# Chapter 6 – Discrete Probability Distributions

## OUTLINE

1. Discrete Random Variables
2. The Binomial Probability Distribution
3. The Poisson Probability Distribution

## Putting It Together

Recall, the probability of an event is the long-term proportion with which the event is observed. That is, if we conduct an experiment 1000 times and observe an outcome 300 times, we estimate that the probability of the outcome is . The more times we conduct the experiment, the more accurate this empirical probability will be. This is the Law of Large Numbers. We can also use counting techniques to obtain theoretical probabilities if the outcomes in the experiment are equally likely. This is called classical probability.

A probability model lists the possible outcomes of a probability experiment and each outcome’s probability. A probability model must satisfy the rules of probability. In particular, all probabilities must be between 0 and 1, inclusive, and the sum of the probabilities must equal 1.

Now we introduce probability models for random variables. A random variable is a numerical measure of the outcome to a probability experiment. So, rather than listing specific outcomes of a probability experiment, such as heads or tails, we might list the number of heads obtained in three flips of a coin. We begin by discussing random variables and describe the distribution of discrete random variables (shape, center, and spread). We then discuss two specific discrete probability distributions: the binomial probability distribution and the Poisson probability distribution.

## Section 6.1 Discrete Random Variable

### Objectives

1. Distinguish between Discrete and Continuous Random Variables
2. Identify Discrete Probability Distributions
3. Graph Discrete Probability Distributions
4. Compute and Interpret the Mean of a Discrete Random Variable
5. Interpret the Mean of a Discrete Random Variable as an Expected Value
6. Compute the Standard Deviation of a Discrete Random Variable

#### Objective 1: Distinguish between Discrete and Continuous Random Variables

Objective 1, Page 1

1. Give the definition of a random variable.

Objective 1, Page 2

1. There are two types of random variables, discrete and continuous. Explain the difference between them.

Objective 1, Page 4

**Example 1 *Distinguishing between Discrete and Continuous Random Variables***

Determine whether the random variable is a discrete random variable or a continuous random variable.

1. The number of as earned in a section of statistics with 15 students enrolled
2. The number of cars that travel through a McDonald’s drive-through in the next hour
3. The speed of the next car that passes a state trooper

#### Objective 2: Identify Discrete Probability Distributions

Objective 2, Page 1

1. Give the definition of a probability distribution.

Objective 2, Page 2

**Example 2 *A Discrete Probability Distribution***

Suppose we ask a basketball player to shoot three free throws. Let the random variable *X* represent the number of shots made; so *x* = 0, 1, 2, or 3. Table 1 shows a probability distribution for the random variable *X*.

**Table 1**

| ***x*** | ***P(x)*** |
| --- | --- |
| 0 | 0.01 |
| 1 | 0.1 |
| 2 | 0.38 |
| 3 | 0.51 |

1. What does the notation *P*(*x*) represent?
2. Explain what *P*(3) = 0.51 represents.

Objective 2, Page 3

1. State the rules for a discrete probability distribution.

Objective 2, Page 5

**Example 3 *Identifying Discrete Probability Distributions***

Which of the following is a discrete probability distribution?

| A) | A) | B) | B) | C) | C) |
| --- | --- | --- | --- | --- | --- |
| *x* | *P(x)* | *x* | *P(x)* | *x* | *P(x)* |
| 0 | 0.16 | 0 | 0.16 | 0 | 0.16 |
| 1 | 0.18 | 1 | 0.18 | 1 | 0.18 |
| 2 | 0.22 | 2 | 0.22 | 2 | 0.22 |
| 3 | 0.10 | 3 | 0.10 | 3 | 0.10 |
| 4 | 0.30 | 4 | 0.30 | 4 | 0.30 |
| 5 | 0.01 | 5 |  | 5 | 0.04 |

#### Objective 3: Graph Discrete Probability Distributions

Objective 3, Page 1

1. In the graph of a discrete probability distribution, what do the horizontal axis and the vertical axis represent?
2. When graphing a discrete probability distribution, how do we emphasize that the data is discrete?

Objective 3, Page 2

**Example 4 *Graph a Discrete Probability Distribution***

Graph the discrete probability distribution given in Table 1.

**Table 1**

| ***x*** | ***P(x)*** |
| --- | --- |
| 0 | 0.01 |
| 1 | 0.10 |
| 2 | 0.38 |
| 3 | 0.51 |

Objective 3, Page 3

Graphs of discrete probability distributions help determine the shape of the distribution.

Recall that we describe distributions as skewed left, skewed right, or symmetric.

#### Objective 4: Compute and Interpret the Mean of a Discrete Random Variable

Objective 4, Page 1

 *Watch the video to learn about the derivation of the formula for finding the mean of a discrete random variable.*

Objective 4, Page 2

1. State the formula for the mean of a discrete random variable.

Objective 4, Page 3

**Example 5 *Computing the Mean of a Discrete Random Variable***

Compute the mean of the discrete probability distribution given in Table 1.

**Table 1**

| ***x*** | ***P(x)*** |
| --- | --- |
| 0 | 0.01 |
| 1 | 0.10 |
| 2 | 0.38 |
| 3 | 0.51 |

Objective 4, Page 4

 *Answer the following after watching the video.*

1. As the number of repetitions of the experiments increases, what does the mean value of the *n* trials approach?
2. As the number of repetitions of the experiments increases, what happens to the difference between the mean outcome and the mean of the probability distribution?

Objective 4, Page 5

**Example 6 *Interpretation of the Mean of a Discrete Random Variable***

The basketball player from Example 2 is asked to shoot three free throws 100 times. Compute the mean number of free throws made.

In each simulation, what value is the graph (that shows the mean number of free throws made) drawn towards?

#### Objective 5: Interpret the Mean of a Discrete Random Variable as an Expected Value

Objective 5, Page 1

Because the mean of a random variable represents what we would expect to happen in the long run, it is also called the expected value, *E*(*X*). The interpretation of the expected value is the same as the interpretation of the mean of a discrete random variable.

Objective 5, Page 2

**Example 7 *Computing the Expected Value of a Discrete Random Variable***

A term life insurance policy will pay a beneficiary a certain sum of money upon the death of the policy holder. These policies have premiums that must be paid annually. Suppose a life insurance company sells a $250,000 one-year term life insurance policy to a 49-year-old female for $530. According to the National Vital Statistics Report, Vol. 47, No. 28, the probability that the female will survive the year is 0.99791. Compute the expected value of this policy to the insurance company.

#### Objective 6: Compute the Standard Deviation of a Discrete Random Variable

Objective 6, Page 1

1. State the formula for computing the standard deviation of a discrete random variable.

Objective 6, Page 2

**Example 8 *Computing the Standard Deviation of a Discrete Random Variable***

Compute the standard deviation of the discrete random variable given in Table 1.

**Table 1**

| ***x*** | ***P(x)*** |
| --- | --- |
| 0 | 0.01 |
| 1 | 0.10 |
| 2 | 0.38 |
| 3 | 0.51 |

Objective 6, Page 4

The variance of the discrete random variable, , is the value under the square root in the computation of the standard deviation.

## Section 6.2 The Binomial Probability Distribution

### Objectives

1. Determine Whether a Probability Experiment is a Binomial Experiment
2. Compute Probabilities of Binomial Experiments
3. Compute the Mean and Standard Deviation of a Binomial Random Variable
4. Graph a Binomial Probability Distribution

#### Objective 1: Determine Whether a Probability Experiment is a Binomial Experiment

Objective 1, Page 1

The binomial probability distribution is a discrete probability distribution that describes probabilities for experiments in which there are two mutually exclusive (disjoint) outcomes. These two outcomes are generally referred to as success (such as making a free throw) and failure (such as missing a free throw). Experiments in which only two outcomes are possible are referred to as binomial experiments, provided that certain criteria are met.

Objective 1, Page 2

 *Answer the following as you watch the video.*

1. What are the four criteria for a binomial experiment?
2. What do *n*, *p*, and  represent when working with a binomial probability distribution?

Objective 1, Page 2 (continued)

1. If *X* is a binomial random variable that denotes the number of successes in *n* independent trials of an experiment, what are the possible values of *X*?

Objective 1, Page 3

**Example 1 *Identifying Binomial Experiments***

Determine which of the following probability experiments qualify as binomial experiments. For those that are binomial experiments, identify the number of trials, probability of success, probability of failure, and possible values of the random variable X.

1. An experiment in which a basketball player who historically makes 80% of his free throws is asked to shoot three free throws and the number of free throws made is recorded.
2. According to a recent Harris Poll, 28% of Americans state that chocolate is their favorite flavor of ice cream. Suppose a simple random sample of size 10 is obtained and the number of Americans who choose chocolate as their favorite ice cream flavor is recorded.
3. A probability experiment in which three cards are drawn from a deck without replacement and the number of aces is recorded.

#### Objective 2: Compute Probabilities of Binomial Experiments

Objective 2, Page 1

 *Watch the video to learn how the binomial probability distribution function is developed.*

Objective 2, Page 2

1. In the formula, what does  represent?
2. In the formula, what do 0.07 and 1 represent?
3. In the formula, what do 0.93 and 3 represent?

Objective 2, Page 3

1. State the Binomial Probability Distribution Function (pdf).

Objective 2, Page 4

1. Fill in the math symbol that is associated with the given phrase.

Phrase math symbol

at least or no less than or greater than or equal to

more than or greater than

fewer than or less than

no more than or at most or less than or equal to

exactly or equals or is

Objective 2, Page 5

**Example 2 *Using the Binomial Probability Distribution Function***

According to CTIA, 55% of all U.S. households are wireless-only households (no landline).

1. What is the probability of obtaining exactly ten wireless-only households based on a random sample of fifteen households?
2. What is the probability of obtaining fewer than three wireless-only households based on a random sample of fifteen households?
3. What is the probability of obtaining at least three wireless-only households based on a random sample of fifteen households?
4. What is the probability of obtaining between five and seven, inclusive, wireless-only households based on a random sample of twenty households?

#### Objective 3: Compute the Mean and Standard Deviation of a Binomial Random Variable

Objective 3, Page 1

1. State the formulas for the mean (or expected value) and standard deviation of a binomial random variable.

Objective 3, Page 2

**Example 3 *Finding the Mean and Standard Deviation of a Binomial Random Variable***

According to CTIA, 55% of all U.S. households are wireless-only households. In a simple random sample of 500 households, determine the mean and standard deviation number of wireless-only households.

#### Objective 4: Graph a Binomial Probability Distribution

Objective 4, Page 1

To graph a binomial probability distribution, first find the probabilities for each possible value of the random variable. Then follow the same approach as was used to graph discrete probability distributions.

Objective 4, Page 2

**Example 4 *Graph a Binomial Probability Distribution***

1. Graph a binomial probability distribution with n = 10 and p = 0.2. Comment on the shape of the distribution.
2. Graph a binomial probability distribution with n = 10 and p = 0.5. Comment on the shape of the distribution.
3. Graph a binomial probability distribution with n = 10 and p = 0.8. Comment on the shape of the distribution.

Objective 4, Page 3

1. What is the shape of the binomial probability distribution if *p* < 0.5, if *p* = 0.5, and if *p* > 0.5?

Objective 4, Page 5

 *Answer the following after Activity 1: The Role of n, the Number of Trials of a Binomial Experiment, on Distribution Shape*

1. As *n* increases, describe what happens to the shape of a binomial probability distribution.

Objective 4, Page 6

1. Under what conditions will a binomial probability distribution be approximately bell-shaped?
2. Explain how to determine if an observation in a binomial experiment is unusual.

Objective 4, Page 7

**Example 5 *Using the Mean, Standard Deviation, and Empirical Rule to Check for Unusual Results in a Binomial Experiment***

According to CTIA, 55% of all U.S. households are wireless-only households. In a simple random sample of 500 households, 301 were wireless-only. Is this result unusual?

## Section 6.3 The Poisson Probability Distribution

### Objectives

1. Determine Whether a Probability Experiment Follows a Poisson Process
2. Compute Probabilities of a Poisson Random Variable
3. Find the Mean and Standard Deviation of a Poisson Random Variable

#### Objective 1: Determine Whether a Probability Experiment Follows a Poisson Process

Objective 1, Page 1

1. For what situations in the Poisson probability distribution used?

Objective 1, Page 2

**Example 1 *Illustrating a Poisson Process***

A McDonald's® manager knows from experience that cars arrive at the drive-through at an average rate of two cars per minute between the hours of 12:00 noon and 1:00 PM. The random variable X, the number of cars that arrive between 12:20 and 12:40, follows a Poisson process.

Objective 1, Page 3

1. Under what conditions does a random variable *X* follow a Poisson process?

#### Objective 2: Compute Probabilities of a Poisson Random Variable

Objective 2, Page 1

1. State the Poisson Probability Distribution Function.

Objective 2, Page 2

**Example 2 *Computing Probabilities of a Poisson Process***

A McDonald's manager knows that cars arrive at the drive-through at the average rate of two cars per minute between the hours of 12 noon and 1:00 PM. Find the following probabilities.

1. Find the probability that exactly six cars arrive between 12 noon and 12:05 PM.
2. Find the probability that fewer than six cars arrive between 12 noon and 12:05 PM.
3. Find the probability that at least six cars arrive between 12 noon and 12:05 PM.

#### Objective 3: Find the Mean and Standard Deviation of a Poisson Random Variable

Objective 3, Page 1

1. State the formula for the mean and standard deviation of a Poisson random variable.

Objective 3, Page 2

1. Restate the Poisson probability distribution function in terms of its mean.

Objective 3, Page 3

**Example 3 *Beetles and the Poisson Distribution***

A biologist performs an experiment in which 2000 Asian beetles are allowed to roam in an enclosed area of 1000 square feet. The area is divided into 200 subsections of 5 square feet each.

1. If the beetles spread themselves evenly throughout the enclosed area, how many beetles would you expect in each subsection?
2. What is the standard deviation of X, the number of beetles in a particular subsection?
3. What is the probability of finding exactly eight beetles in a particular subsection?
4. Would it be unusual to find more than 16 beetles in a particular subsection?